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ABSTRACT

This investigation focuses upon the question of: (1) how naive subjects organize and represent knowledge when solving problems; (2) how previous experience of a novice is used in solving a problem; (3) what kinds of information and knowledge are sought as well as overlooked by novices when solving problems; (4) what kinds of strategies novices employ when solving problems; and (5) what cognitive abilities can be associated with the more successful novice problem solver. The problems selected were based upon a phenomenon about which every individual has some general awareness and common understanding. The problem series presented a sequence of phases, each of which builds upon the previous one. The sequence required the subject to restructure the problem elements previously used in a new and extended environment. Problem sequences were presented to individual subjects via a clinical interview. Once the subject had presented his or her view of the problem phase, it was graphically simulated on the computer screen. The subject then compared his or her interpretations of events with that generated by the computer. This study involved 10 graduate students in a master's level program for elementary education. In general the subjects that made the greatest progress were those that were able to construct, adjust and redefine problem representations. (CW)

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Abstract

KNOWLEDGE REPRESENTATION IN NOVICE PHYSICS PROBLEM SOLVERS

This investigation focuses upon the question of how naive subjects organize and represent knowledge when solving problems. How is the previous experience of a novice used in solving a problem? What kinds of information and knowledge are sought as well as overlooked by novices when solving problems? What kinds of strategies do novices employ when solving problems? What cognitive abilities can be associated with the more successful novice problem solver?

The problems selected for this investigation were based upon phenomenon about which every individual has some general awareness and common understanding. The problems involved objects shot from a gun into space. The problem series presents a sequence of phases each of which builds upon the previous one. The sequence requires the subject to restructure the problem elements previously used in a new and extended environment.

The problem sequence is presented to individual subjects via a clinical interview. Once the subject has presented their view of the problem phase, it is graphically simulated on the computer screen. The subjects then compare their interpretations of events with that generated by the computer. This provides a mechanism for exploring the interactions between the subjects' internal representation and those generated with the computer simulation.

A group of 10 graduate students in a masters level program for elementary education served as subjects. None of the subjects had studied physics in their undergraduate program and were therefore regarded as

novices in the area of projectile motion. The responses of the subject population varied considerably. Some were able to make only limited progress throughout the entire problem sequence. Others were able to successfully complete major portions of the sequence demonstrating mature problem solving abilities.

All subjects responded in a similar manner to the initial problem in which a projectile is shot horizontally out into space. They diagramed a horizontal section with time reference marks indicating an object traveling with uniform motion. This projectile subsequently "arched over" and moved along a diagonal path with increasing velocity until it hit the Earth's surface. Subjects appear to use fragments of information to construct their response.

Three groups of subject responses were identified. The responses of the initial group were fragmented and inconsistent. Differing components of their flight diagrams lacked coordination. Aspects of an object's motion were not related to others. They experienced difficuties making transformations (operations) on the object's paths as seen by observers in different reference frames. Their use of knowledge contained in the simulation was limited. The responses of the second group to the problem sequence were more systematic and consistent. Those subjects recognized and used information from the simulations in addressing subsequent problems. The third group demonstrated better overall problem solving approaches. The simulations appeared to serve as a description useful in monitoring and evaluating their own problem responses. In general the subjects that made the greatest progress with the problem sequence were those that were able to construct, adjust, and refine problem representations.

KNOWLEDGE REPRESENTATION IN NOVICE PHYSICS PROBLEM SOLVERS

Much of the emphasis in the literature regarding expert status has been of limited value in teaching novices (Duda, Heller, Larkin, Lewis, McCloskey). The level of performance that differentiates the expert and the novice is so extreme that it is of limited value in approaching and influencing novice behavior (Chi, Lewis). It may be useful to examine degrees of difference in the manner in which knowledge is processed and organized among novices. In addition it may be beneficial to consider mature novices, individuals that have progressed to some level in the educational system but that have no expertise in the area being investigated. The essential thrust of this investigation focuses upon the question of how naive subjects organize and represent knowledge when solving problems. How is the previous experience a novice used in solving a problem? What kinds of information and knowledge are sought as well as overlooked by novices when solving problems? What cognitive abilities can be associated with the more successful problem solvers?

The problems selected for this investigation were based upon phenomenon about which individuals have some general awareness and understanding. The problems involved objects shot from a gun into space. The problem series presents a sequence of phases each of which builds upon the previous one. The sequence enables

the subject to restructure the problem elements previously used in extended environments. The use of a graduated sequence enables the investigator to follow the problem solving approach of a given subject and to determine at what juncture the subject experiences difficulties. It is then possible to analyze the source of the difficulty of the subject's responses to the previous phases. The difficulties can be expressed in relation to the cognitive abilities and knowledge base of various individuals.

The problem sequence is presented to individual subjects via a structured clinical interview. During the course of the interview the subject is asked to diagram on paper and explain what they think will happen in the problem phase being considered. Once the subject has diagramed on paper their view of the problem phase, it is graphically simulated on the computer screen. The subject can then compare their view of the anticipated events with what is generated by the computer. This provides a mechanism for exploring the interactions between the subject's internal representation and the computer simulations. Subject diagrams and explanations are not themselves regarded as internal representations. These are deeply imbedded in the thought processes of the individual. The diagrams, initial and revised along with explanations and responses to the simulations do, however, provide some indication of the manner in which knowledge is organized and represented internally. A primary consideration is the effect that the simulations have on

subsequent solution attempts of subjects. Since subjects control simulation input, the pattern of data selection also provides some indication of the strategy employed in solving a particular problem phase.

This format provides a number of advantages. No written text is needed to describe the problem. This minimizes decoding and encoding demands and reduces ambiguities. A line segment whose length and direction can be altered in successive iterations provides a simple mechanism by which time-distance relationships can be simulated. The graphics enable the subject to analogically consider various principles and relationships. Solutions can be generated that do not require the use of equations and formalism. The computer further provides direct and immediate feedback. The subject is also provided with various simulation controls. They may change the velocity of a projectile, its direction and position as well as the effects of gravity (on or off). This format provides a mechanism for continually adding information as the sequence develops. Subjects were told that the computer simulation represented one possible solution to the problem. They were asked to compare their solutions with what was generated by the computer. The subjects were advised to compare their own problem solution with the simulations. A small cardboard reference box was used to portray a projection device (gun), projectile, and a target. An observer figure was positioned in the three dimensional space in relation to a projectile and target. The box and its contents are used to

orient the subject and define the problem at various phases of the problem sequence. The position of the observer and objects was altered during the problem sequence.

Subjects

10 graduate students in a masters program in elementary education served as subjects. These individuals were chosen because they had completed a liberal arts undergraduate degree but had not studied physics. They had no formal college level course work in the subject. A few had taken physics while in high school It was assumed that the subjects, as a group, were representative of novice status in relation to physics knowledge. They were mature since they had been admitted in good standing to a visitor's program. Approximately half of the subjects had teaching experience. They all intend to teach science at the elementary school level.

The problem sequence was presented individually and followed a structured protocol. The clinical interviews took approximately 1 1/4 hours. These were taped and subsequently analyzed. The problem sequence was presented to one community college physics teacher and one high school physics teacher with substantial backgrounds in the subject area. These subjects were used to validate the problem sequence. Both subjects completed the sequence in less than half the average time of the elementary school teachers. They encountered only minor difficulties in the latter phase. Their responses provided a reference point as to how an expert would solve the problem

phases.

Problem Sequence

The introductory phase of the problem sequence was designed to establish the background knowledge of the subject as well as their ability to respond to the representational format used in the investigation. Initially a golf ball was held by the investigator several feet above the ground and released. The subject is asked to describe the path of the falling ball and to account for any changes in position and motion. The ball is then rolled along a polished tile floor. The subject is asked to describe this path and to account for any changes in position and motion. Those instances are treated as idealized cases in which no friction is involved. Subjects are then asked to diagram on paper the path of the falling ball. They are told to assume that a clock is ticking at a uniform rate and to record a series of reference marks on the path for successive time intervals. A similar procedure was followed in representing the path of the ball rolling along the floor.

The subject is then asked to draw the path of an object (as seen from the side) shot horizontally out from a given position on a cliff overlooking a plane. Once again they were asked to place reference marks for time intervals along the path and to account for their patterns (Fig. 1). These initial activities provide some indication of the background knowledge of the subjects as well as the extent to which they can separate and coordinate variables in representing time and distance on

their diagrams. Those unable to respond to this format were eliminated as subjects from the investigation.

In the initial phase of the sequence the subject is asked to consider the paths of projectiles shot horizontally out from a given position on a cliff. Different velocities are given to the projectile in subsequent trials. How subjects accommodate the uniform horizontal velocity with the increasing vertical velocity of the projectile is at issue. In subsequent trials the horizontal velocity is increased. After each projectile shot is diagramed, the subject is asked to reveal time reference marks on the projectile path. The particular trial is then simulated on the computer. After a few instances, relationships between the vertical and horizontal components of projectile velocity can be established (Fig. 2).

The subject is then asked to diagram the projectile paths for observers in different positions or reference frames with respect to the projectile path. The initial position (Observer A) was for an observer on the ground to the side of the gun and projectile. The second position (Observer B) was for an observer immediately behind the gun. The third position (Observer C) was for an observer far beneath (or above) the gun and projectile (Fig. 3). In each instance the subject was told that the Earth was to be regarded as composed of transparent material that could easily be seen through. The task was to determine whether the subject could represent the phenomenon from different reference positions. Once the subject had diagramed the paths from

different reference points, time reference marks were added. Each instance was then simulated on the computer.

In the second phase, the projectiles are shot up into space at various angles with respect to the horizontal. In this instance the projectile has an initial vertical component of velocity. The gun is shot (with constant velocity) in successive trials at angles of 70°, 25°, and 80° (Fig. 4). The influence of gravity at the beginning of the projectile flight reduces the vertical component of projectile velocity and increases it after the midpoint. This overall path is influenced by the magnitude of the launch angle. The final question in this phase is at what angle should the gun be aimed to produce the maximum horizontal range.

In the third phase a target object is added to the problem (Fig. 5). The target is positioned just opposite (same height) the gun. The subject is asked to aim the gun so that a projectile will hit (intercept) the target object (released the instant the gun is fired) before the target object itself hits the ground. Both objects are affected by gravity in the same manner. In addition, the projectile has a horizontal velocity. How subjects relate these factors in solving the problem is the primary question in this phase. Variations include their response to the problem when the projectile velocity is changed.

In the fourth phase, the situation is unchanged except that the gun is positioned at ground level, below the target (Fig. 6). The question is the same as in the previous phase. How should

the gun be aimed to hit (intercept) the target object (released the instant the gun is fired) before the target object itself hits the ground.

In the final phase the target is also positioned at ground level and given an initial vertical velocity (Fig. 7). The problem for the subject is to position and fire the gun at a time that will enable the projectile to hit (intercept) the target object before the target object itself hits the ground.

Problem Solutions

A broad range of responses was generated by the subject population. None of the subjects, however, successfully completed the entire problem sequence although a few did make substantial progress through the sequence. One individual repeatedly confused the distance time representations in the introductory phase. The difficulties appeared to be related to the representational format. The results from this subject's interview are, therefore, not included with the data. The other subjects experienced no difficulties with the format or the problems were so minor that they quickly adjusted to the format.

The subjects in the investigation responded similarly to the initial problem phase. When diagramming and describing the motion of a ball shot horizontally off from a cliff, they felt that the ball would initially travel straight out. After a period of time they felt that the ball would 'arch over' and fall to the surface (Fig.8a). A few subjects felt that after arching over the ball would proceed along a diagonal path to the surface

(Fig. 8b). A few others felt the latter part of the path would be straight down. After placing time reference marks on the path, they explained that the ball went out with uniform velocity until it lost some of the velocity. At this point gravity began to take over and pull the ball down with increasing velocity. The subjects seemed reasonably certain that the ball would initially go straight out and that it would eventually fall to the earth. They appeared to be much less certain about what occurred between these end points. They felt that since the gun was aimed straight out, the ball would initially go straight out. After a period of time, something happened to enable gravity to take over and pull the ball down. When questioned about the loss of velocity in a frictionless environment, they argued that either some of the initial motion was used up or that gravity must intervene and slow the ball. In any case they seemed to feel that the forces acting upon the ball should be applied one after the other. Apparently they did not recognize that two or more forces could act simultaneously upon the ball.

As different velocities were applied to the ball, the subjects contended that those shot with greater velocities would travel farther out before turning and moving to the surface. Conversely, objects shot with small velocities would not travel out as far before turning and falling to the surface. There was a reasonableness to this position, particularly in the absence of any perceptual experience. Objects shot out, go out and are eventually pulled down by gravity. There is no experiences that

provide perceptual information about an object's position or path through space. The subjects appeared to construct a position based on information about beginning and end points. The thought that an object might go out as it also goes down apparently had not occurred to these individuals.

The subjects appeared comfortable with the concept of gravity. They tended to regard it as something that pulled objects down to the surface. They felt that the source of gravity was located within the Earth, most likely at the center. When the effect of gravity upon objects of different masses was considered, the limitation of their concept surfaced. Their knowledge of gravity appeared to be more definitional than conceptional.

Three Subject Groups

The responses to the introductory problem phase were similar. The responses to the remaining phases influenced by their interaction with the simulation varied considerably. Although the responses could be viewed as points on a continuum, they have been organized into three groups to facilitate data presentation and to highlight findings. The first group consisted of 4 subjects that experienced difficulties throughout the sequence. The second group consisted of 3 subjects. They made significant progress by making use of information contained in the simulations. The third group consisted of 2 subjects that demonstrated the most mature problem solving strategies. The divisions are based on the overall performance responses to the

problem sequence.

Group I

The 4 subjects in this group experienced difficulties with most phases of the problem sequence. The behavior that most characterize subjects in this group was the limited response to information presented in the computer simulation. Once subjects had diagramed and explained a solution to a problem phase, it was simulated graphically on a computer. Subjects were then asked to compare their solutions to the simulations generated by the computer. Subjects in this group, expressed an initial position about the path of the projectile shot from a gun. After exposure to the entire problem sequence, their problem diagrams remained essentially unchanged. They failed to respond to differences between their paths and those generated in the computer simulations. They maintained that their diagrams were essentially the same as those generated in the graphic simulations. When discrepancies were indicated, they maintained that their diagrams were correct and that the simulation was in error. In some instances they adjusted their diagrams to more closely resemble those presented in the simulation. In subsequent phases, however, they returned to their initial positions in which the projectile is shot straight out at a uniform rate, arches over and falls to the surface with increasing velocity.

In diagraming solutions to subsequent phases, these subjects tended to extend an overall pattern shape. The path shape for an

object shot with velocity four was extended for one shot with velocity 8. The path for the velocity 8 object was drawn with a longer initial segment. They used the same general path shape for an object shot with velocity 100 (Fig. 9). In the restricted area of the computer screen this simulated path appears essentially as a straight line. When representing the path of projectiles they used an overall shape pattern that was modified slightly from situation to situation. They failed to adjust the shape of the path in relation to parameters that differed significantly from those they used initially.

Although the subjects maintained a uniform overall path shape for projectiles shot with different velocities, they predicted different flight times for these objects. Projectiles shot with a velocity of 4 or 8 would be in the air for the same length of time. They argued, however, that projectiles shot with velocity 100, however, would be in the air for a longer period of time since it had a greater distance to travel. Situations that used similar velocity values (velocity 4, velocity 8) were viewed similarly. Situations that presented significantly different velocity values were viewed problems requiring different solutions.

Similar examples occurred throughout the sequence. In the target phase for example, subjects learned that a projectile fired with sufficient velocity directly at a target (at the same height as the gun) would hit the target object before it reached the ground. If a projectile were then fired with a very small

velocity, these individuals felt that the path would have the same overall shape but would not hit the target regardless of how long a time or how deep a fall the target might make. They argued that after a few intervals, the projectile would begin to fall so rapidly in a vertical direction that it would never hit the target (Fig.10). These subjects appeared to generate an overall representation for the problem, but they did not appear to reason closely with or from it. Their responses appeared to be influenced by the most conspicuous or outstanding features contained in the graphic simulations. Their visual representation and reasoning did not appear to be well coordinated

Pattern Recognition

When Group I subjects compared reference marks they placed on projectile paths with those made with the computer simulations, they did not recognize or respond to differences or similarities in the vertical and horizontal components of simulated motion for projectiles shot with varying velocities. In time they did recognize that the vertical components were the same for different shots. They, however, failed to recognize the uniformity of the horizontal components, both for any given shot or for shots with differing velocities. Furthermore, any pattern that might have been recognized was not included within the paths they diagramed in subsequent shots (Fig. 11). When placing reference marks on the projectile paths, they continued to add them sequentially, first along the horizontal section of the path

and then along the diagonal. They did not appear to respond to the two dimensional aspects of the projectile's motion as viewed from the side.

Transformations

In the initial section of the problem sequence, the phenomenon is considered from the vantage point of an observer (Observer A) to the side of the projectile path. Subjects are asked to consider how an observer (Observer B) would report the same projectile flight from a position behind the gun, and then how an observer (Observer C) would report the same projectile flight from a position far underneath (or overhead). The subject was asked to imagine that the observers were all connected by telephone and could compare observations. Group I subjects experienced difficulties representing the projectile paths as seen by different observers. In fact they indicated that Observer B and Observer C would report paths much like that reported by Observer A (Fig.12). When they examined the computer simulation, they seemed puzzled. After viewing a number of simulations, they began to generate patterns that more closely resembled those found in the simulations. They did not, however, incorporate this information into subsequent trial shots. They also maintained that the path lengths for the three observers were different. They further indicated that the times of flight as observed by the three observers were also different. An object shot from the cliff would have different path lengths and be in the air for different periods of time. Their knowledge

about the characteristics of an object's flight appeared fragmentary. What Observer A reported appeared to have limited relationship to what Observer B or Observer C would report. The event for these subjects appeared to be different for each observer, at least as far as path length and time of flight were concerned.

Altered Situation

In an intermediate phase, the projectile was shot upward at some specified angle with respect to the horizontal. The intent was to provide the projectile with some vertical component of velocity that gravity would counter. The influence of gravity on the object would differ from the previous situations where no vertical component of velocity was involved initially. Subjects approached this problem in a manner similar to that used previously. They felt that an object shot upward would start out slowly and increase in velocity as it rose up along the path. They maintained this position until they recognized some internal conflict with this position. When the object reached the apex of its path, they realized that it should start to fall back to earth. They argued however, that the velocity was increasing at that point. They then recognized the contradiction and altered their position. The object started out with a given value that lost velocity on the way up. When the velocity reached zero, it started to fall back to the surface. It is noteworthy that these subjects recognized that gravity influenced the motion of a ball the instant it left the gun. This contrasts with their view that

gravity does not effect the path of a ball shot horizontally out from the gun until some of its initial velocity had been lost whereupon gravity "took over" and pulled the object down.

Surface Features and Relationships

Projectiles were launched (in subsequent trials) at constant velocity but at different angles with respect to the horizontal. Initially the gun was aimed at an angle of 70° the 25°, and finally at 80°. Although the overall shape of the paths was maintained, the subjects did not coordinate vertical and horizontal dimensions of the paths as the aiming angles were changed. They did not recognize that as the aiming angle decreased toward zero, the horizontal components of motion increased and the vertical components decreased.

When the aiming angle approached zero, they felt that the range (horizontal distance) should be at a maximum value. They appeared surprised in the simulation to find that a projectile shot at this angle had essentially no range. These subjects then suggested some angle between 0° and 25° be used to aim the gun to produce the maximum range. The 25° was the previous value they had used. They tried several values for angles within this interval. They only referred to the aim angle and the range distance produced. They did not examine underlying relationships such as might exist between the aiming angle and the length of time that the projectile was in flight. They appeared to attend to the surface features of the problems as represented by the numerical values they failed to examine

underlying relationships that the numerical values might express.

Data Organization

Their use of data appeared to be essentially random. In aiming the gun to find maximum range, they tried 5°, then 10°; then 15° etc. They tried one value after another. These values appeared to be random. When shooting the gun (with sufficient velocity) at a target directly opposite (target released the instant the gun is fired), they would aim the gun above or below the horizontal. After viewing a trial on the computer, they would then use successively smaller values. For example 30°, 20°, 10°, 5°, 3°, 2°, 1°, 1/2° was one pattern used. They avoided a zero value and used successively smaller values that approached zero. They appeared not to recognize any relationships that might be involved in the problem. They chose values that generated a result that brought them closer to the desired end product. They were unable to explain why the changes in angle produced closer collisions. Their strategy consisted of manipulating surface features on a trial and error basis that produced a desired result. They also tended to extend a sequence of trial changes that approached a limiting value. This strategy proved to be essentially useless in solving the remaining phases in the problem sequence.

Group II

The second group consisted of three subjects whose responses were similar to one another but different from others in the overall subject group. The most distinguishing feature of this

groups' responses to the problem sequence was their reaction to and the computer simulations. They, initially, as did individuals in Group I, felt that a projectile shot horizontally from the gun would travel out, arch over, and fall diagonally to the ground. They, however, recognized discrepancies between their representation and the computer simulations. The subjects appeared to check their position with the information contained in the simulation. One subject recognized in the simulation that the projectile started to move vertically from the moment of launch. This individual said, "At first, I thought about this incorrectly. Gravity pulls the object down from the very beginning." They recognized that gravity was operational continuously and, therefore, must influence the path of projectile from the moment it left the gun. The subjects in this group were able to recognize and use information contained in the simulation in solving subsequent problems.

When generating the path of a projectile shot with an unusually large velocity (velocity 100), they diagramed a path that was close to a straight line. This shape differed significantly from the path shapes of objects with velocity 4 and velocity 8. Their construction was accommodated to the problem parameters. They recognized that a slight vertical component was necessary for the limited time period that the projected object would remain on the screen. When asked which of two objects shot systematically (one with velocity 4 and one with velocity 100) would hit the ground first, they recognized that both hit the

ground at the same time. Both objects fell the same vertical distance at the same rate.

These subjects were better able to recognize patterns in the vertical and horizontal velocity components of projectiles shot with differing velocities. They were also better able to diagram the path of projectiles as seen by observers in different reference positions (Observer A, Observer B, Observer C). These subjects like those in Group I were still, however, unable to effectively coordinate the information represented by the different observers.

Subjects in Group II approached the target phase of the sequence in a manner similar to those in Group I. When aiming the gun to hit the target with a projectile, they positioned the gun either above or below the line of sight to the target. These subjects did follow, however, the path of the projectile as it moved to intercept the target. They observed where and when it crossed the target path. They chose the angle for the next shot on the basis of where the projectile and target paths crossed. The magnitude of the adjustment angle was influenced by the nearness of collision. Their choices did not appear to be random as did those in the initial group.

The subjects in this group appeared to respond to information contained in the simulations. They appeared to use information in the simulation to monitor their own responses. The simulation would present solution patterns that differed from their own. They would use the information presented in the

simulation to check the assumptions and relationships they had made. The simulations were used to monitor their solution approaches.

The Third Group

The third group consisted of two individuals. They experienced difficulties similar to the others in the early phases of the problem sequence. They are identified here because they were able to progress further in the problem sequence. The strategies they used in solving the problems also differed. This was apparent in the latter part of the second phase (aiming phase). One subject pointed out that as the gun aimed at a lower angle, its horizontal component increased as its vertical component decreased. It, therefore, goes out with greater velocity but is not in the air as long. Conversely as the gun was aimed at greater angles, the speed with which the projectile went out horizontally was smaller although it was in the air for a longer period of time. The subject explained that "diminishing returns" existed at either end and that the best place for aiming was somewhere near the middle where you took advantage of both parts. The subject suggested trying 45°.

In the target phase, these subjects responded to the question of how to shoot the projectile so that it would hit the target while in the air, with the statement that the gun should be aimed directly at the target. In their diagrams they indicated that the target fell down and the projectile also fell down as it moved out towards the target. When the velocity was

increased the subject said the gun should still be aimed at the target but it would hit the target sooner. The opposite condition would hold for a decrease in velocity. If the velocity was too small it would hit the ground before making contact with the target.

In the next phase where the gun is positioned (at ground level) below the target, the two subjects experienced some initial difficulties. They used drawings to represent salient and minimal features of the two objects and their interaction. A diagram of the falling target was drawn first and then the projectile's path was added. One individual suggested that gravity should be turned off and the gun pointed directly at the object. Once gravity is turned on both objects are influenced identically. The gun, therefore, should still be pointed directly at the target. The objects must collide since they are in the air for the same time and are pulled down the same amount.

In the final phase when the projectile and target are located at ground level and the target is given an initial vertical thrust upward, the subject is given the option of aiming the gun and shooting the gun (at a time of their choice). One subject diagramed the target being shot up (with reference marks). Then the subject drew the projectile path. The subject explained that if you wait until the target reached its apex, you can shoot the gun directly at it. At this point, the problem is exactly the same as the previous problem.

The two subjects in this group appeared to use information

provided in the simulation. They also sought additional information. What would it be like if gravity was not present? In addition they used the diagrams to represent the most salient features of the problem. If there were two objects, they would diagram both and try to correlate aspects of the objects behavior. When aiming at different angles or shooting with different velocities, they appeared to have generated some ideas and would suggest a particular instance which served as a test probe. They abstracted principal features of the problem and tried to relate their behavior in relation to common influences. These individuals appeared to use their representations to facilitate their reasoning.

Summary

A group of graduate students in a masters level program for elementary education served as subjects. None of the subjects had studied physics in their undergraduate program and were therefore regarded as novices in the area of projectile motion. The responses of the subject population varied considerable. Some were able to make only limited progress throughout the entire problem sequence. Others were able to successfully complete major portions of the sequence demonstrating mature problem solving abilities. Differences in individual reactions to the sequence and of responses to the simulations provided an indication of the more promising problem solving approaches.

All subjects responded in a similar manner to the initial projectile problem by diagraming a horizontal section with

reference marks of an object traveling with uniform motion. This projectile subsequently arched over and moved with increasing velocity until it hit the surface. Such responses were undoubtedly related to the limited nature of common experience. These subjects were unable to perceive the changes in motion of a falling ball held at arm's length and released. The path and changes in motion of an object shot into space are at least as difficult to perceive. Subjects apparently used pieces of knowledge to construct their responses. They apparently assumed that an object shot straight out would go straight out. After being in free space for a period of time, they knew it would fall down to the surface. They had only limited knowledge of what might occur during the mid-portion of the trajectory. They claim that some of the initial thrust of the object must have been used up. Gravity then took over and pulled the object down to the surface. In such instances, they have no direct experience with which to evaluate the appropriateness of their position. In this investigation the computer simulations provided this function.

Three groups of subject responses were identified. The responses of the initial group were fragmentary and inconsistent. One aspect of an object's motion was not coordinated to another. The subjects had difficulty making transformations (operations) on the object's paths as seen by observers in different reference frames. Their use of knowledge presented in the simulation was limited. They were unable to extract and use information in

subsequent phases. On the contrary, they tended to select and use information incorrectly. Specific choices were influenced by proximity in time and context. Their choices and use of data appeared to be essentially random. These subjects rarely sought additional information nor did they try to reduce the overall problem complexity. The strategies they used in solving problems appeared limited.

The responses of the second group to the problem sequence were more systematic and consistent. They made substantial progress in the problem sequence. Their problem diagrams were better coordinated and they recognized and used information from the simulations in addressing subsequent problems. They appeared to use information presented in the simulations to monitor their problem responses. The greatest difficulties occurred for them when a second object (target) was introduced.

The third group demonstrated better overall problem solving approaches. They coordinated component parts with the overall path of the projectile. They also extracted and used information from the simulation in subsequent phases. They were also able to make transformations (operations) on their diagrams with facility. Their diagrams were used to isolate and reference salient features of problem objects. When they encountered difficulties, they sought additional information and attempted to reformulate the problem. They appeared to use the simulation to monitor and debug their approaches to the problems. The simulations were also recognized as providing opportunities for

initiating test probes for evaluating alternative problem solutions.

Discussion

In general the subjects that made the greatest progress in the sequence were those that were able to use the simulations in monitoring their own problem solving approaches. Their representations appeared to be more integrated and more abstract. They tended to consider phenomena such as the object path as a whole or Gestalt rather than as separate component parts. Their representations were more abstract in the sense that the more salient features of the problem situation were presented. A ball was drawn as a dot or a projectile as an arrow. Their representations apparently enabled them to recognize relationships involved in the problem as well as to better analyze and respond to various aspects of the simulations.

The individuals that demonstrated better cognitive skills made the greatest overall progress in the sequence. These individuals were able to consider various aspects of a problem situation from different perspectives. This flexibility appeared to be related to their ability to restructure or reorder relationships in a problem situation. These skills were also related to their ability to generate and extend patterns from a set of observations or data points. These individuals were able to look beyond the values of particular instances to consider underlying relationships. Their representations appear to serve as an integrating mechanism by which the individual can monitor

and evaluate their problem solving approaches.

Various cognitive abilities appear to be associated with well integrated abstract representational systems. These abilities appear to be closely associated with generating, restructuring, and refining problem representations. The individuals that have developed these abilities appear to be on the way to becoming successful problem solvers. The individuals that demonstrated a greater range of cognitive abilities, although classified as novices in relation to their knowledge base, made significant progress toward higher levels of problem solving in projectile motion. Those most lacking in these abilities experienced difficulties throughout the sequence. Cognitive abilities related to organizing and processing information in relation to representation appear to be the factors that differentiate the most successful problem solvers.

Individuals unable to generate and alter their representations are bound to the specificity of reality. They focus primarily upon the surface features of objects. They also fail to recognize patterns and underlying relationships. They can only select random values or those with some proximity to the question at hand. They also extend a pattern of values with the expectation that meaning resides within the sequence of values particularly as they approach some critical or limiting value. Their ability to solve problems would appear to be seriously restricted.

Implications for Teaching

A number of implications for science teaching are suggested as a result of this investigation. These occur at different levels. In the first place, the continuing emphasis upon expository teaching raises questions. The understanding of fundamental concepts of science appear limited with many students. The orientation to phenomena such as gravity appear to be more definitional than conceptual. It can be highly specific and useful in only the most restricted instances.

Models of phenomena are frequently imposed. This denies students opportunities to develop abilities for abstracting and representing such phenomena. The subjects that were able to generate flexible problem representations made the greatest progress in the problem sequence. It would follow that the development of cognitive abilities associated with generating and operating from a representation would be of increasing interest in science education. Rather than impose models of various phenomena, students should be encouraged to generate and evaluate their own models of phenomena. The teacher's attention could then be directed at developing the abilities associated with organizing and processing knowledge.

The role of experience in concept development is recognized as being critical. The use of computer simulations could appear to offer a number of opportunities. Simulations are dynamic and provide access to phenomena not easily presented in texts, teacher presentations, or even in laboratory experiments. The response is direct and the feedback is immediate. The

simulations also enable the student to extend reality. They can turn gravity off and alter phenomena in ways not possible in reality. The use of graphics also enables the teacher to present phenomena metaphorically, deferring the use of equations and formalism until the student is sufficiently mature. Simulations also provide another representational system that students can react to in developing understanding.

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